

Nondifferentiable Dynamic: Two Examples

V. Dzhunushaliev *

Institut für Mathematik, Universität Potsdam PF 601553, D-14415 Potsdam, Germany

Some nondifferentiable quantities (for example, the metric signature) can be the independent physical degrees of freedom. It is supposed that in quantum gravity these degrees of freedom can fluctuate. Two examples of such quantum fluctuation are considered: a quantum interchange of the sign of two components of the 5D metric and a quantum fluctuation between Euclidean and Lorentzian metrics. The first case leads to a spin-like structure on the throat of composite wormhole and to a possible inner structure of the string. The second case leads to a quantum birth of the non-singular Euclidean Universe with frozen 5th dimension. The probability for such quantum fluctuations is connected with an algorithmical complexity of the Einstein equations.

I. INTRODUCTION

It is possible that some discrete mathematical objects which can not be the continuous functions nevertheless are the physical degrees of freedom in the Nature. It can be a signature of metric, dimensionality, topology of space and so on. A time evolution of such kind of the variables is a big problem for the classical and quantum gravity.

The change of the metric signature in the classical gravity ordinary is connected with the presence of a surface δ -like matter (see for example, [1], [2]). Certainly, we have the question: is this matter exotic or ordinary, i.e. can such conditions realized in the Nature ? In quantum gravity the change of metric signature is the result of integrating in the path integral over the Euclidean and Lorentzian metrics. The difficulties connected with this problem is easy to see in the vier-bein formalism

$$ds^2 = \eta_{ab} e^a e^b \quad (1)$$

here $\eta_{ab} = (-, +, +, +)$, $e^a = h_\mu^a dx^\mu$, $a = 0, 1, 2, 3$ is the vier-bein index, μ is the spacetime index. In the classical regime only the tetrad components h_μ^a is the dynamical variables and varying with respect to h_μ^a leads to the Einstein equations. But we can not vary with respect to η_{ab} and therefore we have not the corresponding equations. This allow us to say that the difficulties connected with the signature change are connected with that η_{ab} are the nondynamical variables. In quantum gravity η_{ab} become the dynamical quantities. We see that η_{ab} are the discrete variables and in fact an integration over η_{ab} should be a summation.

*E-Mail Addresses : dzhun@rz.uni-potsdam.de and dzhun@freenet.bishkek.su; permanent address: Dept. Theor. Phys., Kyrgyz State National University, Bishkek 720024, Kyrgyzstan

In this paper we would like to show that in quantum gravity can exist some discrete (nondifferentiable) physical degrees of freedom ¹ which also can be the dynamical variables. In particular the signature change can happen not on the boundary between different regions with the Euclidean and Lorentzian regions but it can take place a fluctuation (“quantum trembling”) between $\eta_{ab} = +1$ and $\eta_{ab} = -1$.

II. FLUCTUATION WITHOUT THE CHANGE OF THE SIGN OF METRIC DETERMINANT

Let we consider the 5D metric

$$ds^2 = -\sigma\Delta(r)dt^2 + dr^2 + a(r)d\Omega^2 + \sigma\frac{r_1^2}{\Delta(r)}(d\chi - \omega(r)dt)^2 \quad (2)$$

here χ is the 5th extra coordinate; r, θ, φ are the 3D polar coordinates; t is the time; $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the metric on the S^2 sphere; $\sigma = \pm 1$ describes the interchange of metric signature: $(-, +, +, +, +) \leftrightarrow (+, +, +, +, -)$. The functions $\Delta(r), a(r)$ are the even functions this means that the 3D part of the (2) metric is the wormhole-like 3D space. The 5D vacuum Einstein equations give us

$$\frac{\Delta''}{\Delta} - \frac{\Delta'^2}{\Delta^2} + \frac{a'\Delta'}{a\Delta} - \frac{r_0^2}{\Delta^2}\omega'^2 = 0, \quad (3)$$

$$\omega'' - 2\omega'\frac{\Delta'}{\Delta} + \omega'\frac{a'}{a} = 0, \quad (4)$$

$$\frac{\Delta'^2}{\Delta^2} + \frac{4}{a} - \frac{a'^2}{a^2} - \frac{r_0^2}{\Delta^2}\omega'^2 = 0, \quad (5)$$

$$a'' - 2 = 0. \quad (6)$$

with the following solution [3]

$$a = r_0^2 + r^2, \quad (7)$$

$$\Delta = \frac{2r_0}{q} \frac{r^2 + r_0^2}{r^2 - r_0^2}, \quad (8)$$

$$\omega = \frac{4r_0^2}{r_1 q} \frac{r}{r^2 - r_0^2}. \quad (9)$$

here $r_0 > 0$ and q are some constants. We see that the (3)-(5) equations do not depend on the σ . It is the most important thing for understanding as occurs a quantum fluctuation of the metric signature. We have one solution for two metrics with the different signature and the classical dynamical equations (3)-(5) can not distinguish them. But the quantum paradigm says us: *that which is not forbidden is permitted*. Following this rule we can say that in this situation should exist the fluctuations (“quantum trembling”) between two signatures.

¹here we consider the case with the components of η_{ab}

Let η_1 is a quantum state with the $(-, +, +, +, +)$ signature and η_2 with the $(+, +, +, +, -)$ signature. Then we can assume that

$$\eta_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \eta_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (10)$$

as the probabilities for both signatures $(\pm, +, +, +, \mp)$ should be equal. The eigenstates $\eta_{1,2}$ describe the states with $(\mp, +, +, +, \pm)$ accordingly.

We see that the eigenstates (10) are the same as the eigenstates of z -component of the spin

$$\left(\frac{\hbar}{2} \sigma_3 \right) \zeta = \pm \frac{\hbar}{2} \zeta \quad (11)$$

$$\zeta_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \zeta_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (12)$$

here $\frac{\hbar}{2} \sigma_3$ is the operator of z -component of the spin, $\zeta_{1,2}$ are its eigenstates and $\pm \frac{\hbar}{2}$ are its eigenvalues.

This allow us to presume that such “quantum trembling” between two signatures exists and have the physical interpretation in the following sense. In [4] it was proposed a model of composite wormhole consisting from a 5D throat ² and two Reissner-Nordström black holes attached to the 5D throat on the event horizon, see Fig.1.

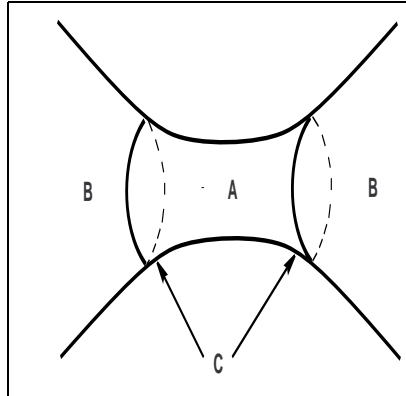


FIG. 1. The composite wormhole: whole space is 5 dimensional but in **B** regions the G_{55} metric component is non-dynamical (in this case 5D gravity is completely equivalent to 4D gravity + electromagnetism), **A** is the 5D throat where G_{55} is the dynamical variable, **C** are the event horizons.

In Ref. [5] Wheeler assumed that the geometrical origin of spin probably is connected with a quantum fluctuation orientability \leftrightarrow nonorientability ³: “If however both spaces ⁴

²which is a solution (7)-(9)

³i.e. we have a “two-valuedness”

⁴orientable and non-orientable

are permissible, then a space with one wormhole has one classical two-valuedness associated with it, and one with n wormholes has n -fold duplicity. If it should turn out that there are 2^n inequivalent ways to get to a geometry with n wormholes, and if it should make sense to assign 2^n distinct probability amplitudes to the same macroscopic field configuration, then one would be in possession of a non-classical two valuedness with as many spin-like degrees of freedom as there are wormholes. ... A number of options shows up which is qualitatively of the same order as the number of degrees of freedom of a spinor field, when one goes to the virtual foam-like space of quantum geometrodynamics. It is difficult to say anything more specific about the reasonableness or unreasonableness of this conceivable "correlation of spin with parity" until more is known about the formalism of quantum geometrodynamics". Here we offer the above-mentioned "unclassical two-valuedness" connected with "quantum trembling" between two classical solutions with the different metric signature as a model of an inner geometrical structure of spin in spirit of the Wheeler idea "spin = two-valuedness".

It is very interesting that our composite WH in some approximation is close to a string attached to two D-branes. Let us factorize the 5th throat by the manner represented on the Fig.2, i.e. all S^2 spheres in the left side of the picture are contracted to the points on the right side, two event horizons are contracted to two points of attachment the string to the branes.

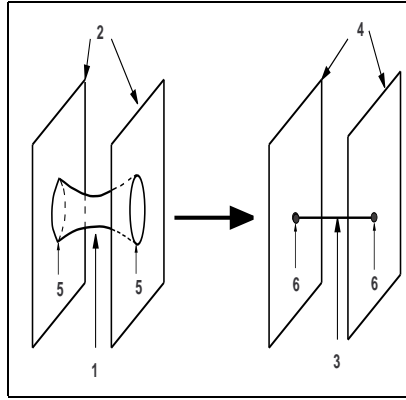


FIG. 2. The factorization of the 5D throat-(1) of composite WH by S^2 spheres leads to a string-like object-(3) attached to two branes-(4), (2) are two black holes. The event horizons-(5) become the points-(6) of attaching the string to D-branes. The throat of composite WH with the flux of electric field after factorization is the string with the flux of electric field and the electric charges at the ends of string. The quantum fluctuation $(-, +, +, +, +) \leftrightarrow (+, +, +, +, -)$ leads to the appearance of a spin-like (super) structure on the string.

As we saw above "quantum trembling" between signatures leads to the appearance of the spin-like structure, i.e. to the fermion degrees of freedom. But it is well known that these degrees of freedom are connected with the grassmanian coordinates. It is possible that this means that the quantum fluctuations between two metric signatures leads to the appearance of a superspace with the ordinary and grassmanian coordinates. In this case the throat of composite WH + fluctuation between two signatures after above-mentioned factorization is equivalent to a superstring attached to two branes, see Fig.2. The whole composite WH is equivalent to the D-brane (superstring + two branes). It allow us to assume that the superstring possible has an inner structure and in this sense it is an approximation for the

composite WH. In this connection it can mention the citation from Ref. [6]: “Discrete degrees of freedom often manifest themselves as fermion in the quantum formalism. It is also conceivable that the continuum theories at the basis of our considerations will have up include string- and D-brane degrees of freedom ...”.

III. FLUCTUATION WITH THE CHANGE OF THE SIGN OF METRIC DETERMINANT

In the previous section we consider the case of the quantum fluctuation between two signatures $(-, +, +, +, +) \leftrightarrow (+, +, +, +, -)$ without the change of the sign of metric determinant. Here we assume that can be a situation when a fluctuation between the Euclidean and Lorentzian metrics is possible. Evidently this can be only a cosmological solution in contrast with the previous spherically symmetric case. This idea differs from the initial Hawking idea about changing of the metric signature on the boundary between Euclidean and Lorentzian regions in such a manner that in the Universe is a region where takes place a quantum fluctuation (“trembling”) between different metric signature. It can be in the very Early Universe on the level of Planck scale.

For example, we examine a vacuum 5D Universe with the metric

$$ds^2 = \sigma dt^2 + b(t) (d\xi + \cos \theta d\varphi)^2 + a(r) d\Omega_2^2 + r_0^2 e^{2\psi(t)} [d\chi - \omega(t) (d\xi + \cos \theta d\varphi)]^2 \quad (13)$$

here $\sigma = \pm 1$ for the Euclidean and Lorentzian signatures respectively. 3D space metric $dl^2 = b(t) (d\xi + \cos \theta d\varphi)^2 + a(r) d\Omega_2^2$ describes the Hopf bundle with the S^1 fibre over the S^2 base. In the 5-bein formalism we have

$$ds^2 = \eta_{\bar{A}\bar{B}} e^{\bar{A}} e^{\bar{B}} \quad (14)$$

here \bar{A}, \bar{B} are the 5-bein indexes and

$$\eta_{\bar{A}\bar{B}} = (\pm 1, +1, +1, +1, +1), \quad (15)$$

$$e^{\bar{0}} = dt, \quad (16)$$

$$e^{\bar{1}} = \sqrt{b} (d\xi + \cos \theta d\varphi), \quad (17)$$

$$e^{\bar{2}} = \sqrt{a} d\theta, \quad (18)$$

$$e^{\bar{3}} = \sqrt{a} \sin \theta d\varphi, \quad (19)$$

$$e^{\bar{5}} = r_0 e^{\psi} [d\chi - \omega(t) (d\xi + \cos \theta d\varphi)] \quad (20)$$

According to the following theorem [7], [8]:

Let G be a structural group of the principal bundle. Then there is a one-to-one correspondence between the G -invariant metrics

$$ds^2 = \varphi(x^\alpha) \Sigma^a \Sigma_a + g_{\mu\nu}(x^\alpha) dx^\mu dx^\nu \quad (21)$$

on the total space \mathcal{X} and the triples $(g_{\mu\nu}, A_\mu^a, \varphi)$. Here $g_{\mu\nu}$ is the 4D Einstein's pseudo-Riemannian metric on the base; A_μ^a are the gauge fields of the group G (the nondiagonal

components of the multidimensional metric); $\varphi\gamma_{ab}$ is the symmetric metric on the fibre ($\Sigma^a = \sigma^a + A_\mu^a(x^\alpha)dx^\mu$, $\Sigma_a = \gamma_{ab}\Sigma^b$, $\gamma_{ab} = \delta_{ab}$; $a = 5, \dots, \dim G$ is the index on the fibre and $\mu = 0, 1, 2, 3$ is the index on the base).

we have the electromagnetic potential

$$A = \omega(t) (d\xi + \cos\theta d\varphi) = \frac{\omega}{\sqrt{b}} e^{\bar{1}} \quad (22)$$

For this potential the Maxwell tensor is

$$F = dA = \frac{\dot{\omega}}{\sqrt{b}} e^{\bar{0}} \wedge e^{\bar{1}} - \frac{\omega}{a} e^{\bar{2}} \wedge e^{\bar{3}} \quad (23)$$

Therefore we have the electrical field

$$E_{\bar{1}} = F_{\bar{0}\bar{1}} = \frac{\dot{\omega}}{\sqrt{b}} \quad (24)$$

and the magnetic field

$$H_{\bar{1}} = \frac{1}{2} \epsilon_{1\bar{j}\bar{k}} F^{\bar{j}\bar{k}} = -\frac{\omega}{a} \quad (25)$$

Let we write down the vacuum 5D Einstein equations

$$R_{\bar{0}\bar{0}} \propto \ddot{\psi} + \dot{\psi}^2 + \frac{\ddot{a}}{a} + \frac{\ddot{b}}{2b} - \frac{\dot{b}^2}{4b^2} - \frac{\dot{a}^2}{2a^2} + \frac{r_0^2}{2b} e^{2\psi} \dot{\omega}^2 = 0, \quad (26)$$

$$R_{\bar{1}\bar{1}} \propto \frac{\ddot{b}}{b} + \frac{\dot{b}}{b} \dot{\psi} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{b}^2}{2b^2} - \sigma \frac{b}{a^2} + \frac{r_0^2}{b} e^{2\psi} \dot{\omega}^2 = 0, \quad (27)$$

$$R_{\bar{2}\bar{2}} = R_{\bar{3}\bar{3}} \propto \frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}}{a} \dot{\psi} + \sigma \left(-\frac{2}{a} + \frac{b}{a^2} + \frac{r_0^2}{a^2} e^{2\psi} \omega^2 \right) = 0, \quad (28)$$

$$R_{\bar{5}\bar{5}} \propto \ddot{\psi} + \dot{\psi}^2 + \frac{\dot{a}}{a} \dot{\psi} + \frac{\dot{b}}{2b} \dot{\psi} + \frac{r_0^2}{2} e^{2\psi} \left(\sigma \frac{\omega^2}{a^2} + \frac{\dot{\omega}^2}{b} \right) = 0, \quad (29)$$

$$R_{\bar{2}\bar{5}} \propto \ddot{\omega} + \dot{\omega} \left(\frac{\dot{a}}{a} - \frac{\dot{b}}{2b} + 3\dot{\psi} \right) - \sigma \frac{b}{a^2} \omega = 0 \quad (30)$$

Now we can formulate our basic assumption: *by some conditions, in one region* ⁵, *can exist a quantum fluctuation between the Euclidean and Lorentzian metric signatures*. This means that in the classical equations (26)-(30) arises a quantum fluctuating quantity σ defining the metric signature. Another words, we have two copies of the classical equations: one with $\sigma = +1$ and another with $\sigma = -1$. The equation (26) is invariant relative to $\sigma = \pm 1$ exchange. Let we consider the remaining equations with σ . The basic question arising in this situation is: how is a probability for each equation with $\sigma = +1$ and $\sigma = -1$ in the (27)-(30) equations system ?

⁵for example, in the very Early Universe

As in Ref. [9] we will define this probability starting from the algorithmical complexity (AC) of each equation. What is the AC and how is its physical interpretation? In 60th Kolmogorov have defined the notion of probability from the algorithmical point of view [10]. His basic idea is very simple: a probability of an appearance of some object depends from its AC and the AC is defined as the *minimal* length of an algorithm describing given object on some universal computer (Turing machine, for example). Simply speaking: *the simpler the more probable*.

The key word for such definition of the probability is the word “minimal”. In this case the length of the algorithm is determined uniquely. The exact definition is [10]

The algorithmic complexity $K(x | y)$ of the object x by given object y is the minimal length of the “program” P that is written as a sequence of the zeros and unities which allows us to construct x having y :

$$K(x | y) = \min_{A(P,y)=x} l(P) \quad (31)$$

where $l(P)$ is length of the program P ; $A(P, y)$ is the algorithm calculating object x , using the program P , when the object y is given.

In this connection we can recall that ‘t Hooft in Ref. [11] has proposed to investigate the Universe as a certain computer: “The finiteness of entropy of a black hole implies that the number of bits information that can be stored there is finite and determined by the area of its horizon. This gave us the idea that Nature at the Planck scale is an information processing machine like a computer, or more precisely, a cellular automaton”.

Now we can presuppose that the fluctuations of the metric signature occurs as the fluctuations between the algorithms (Einstein equations with the different metric signature)

$$\begin{array}{ccc} \sigma = +1 & \longleftrightarrow & \sigma = -1 \\ & \updownarrow & \\ R_{00}^+ & \longleftrightarrow & R_{00}^- \\ R_{11}^+ & \longleftrightarrow & R_{11}^- \\ R_{22}^+ & \longleftrightarrow & R_{22}^- \\ R_{33}^+ & \longleftrightarrow & R_{33}^- \\ R_{55}^+ & \longleftrightarrow & R_{55}^- \end{array} \quad (32)$$

The signs \pm denote the belonging of the appropriate equation to the Euclidean or Lorentzian mode. The expression (32) designates that the appearance of the quantum magnitude σ leads to a quantum fluctuation $R_{AB}^+ \leftrightarrow R_{AB}^-$. The question is: how we can calculate a probability for each R_{AB}^\pm equation? Our assumption for these calculations is that *these probabilities are connected with the AC of each equation*.

A. Fluctuation $R_{25}^+ \longleftrightarrow R_{25}^-$.

The R_{25} equation in the Euclidean mode is

$$\ddot{\omega} + \dot{\omega} \left(\frac{\dot{a}}{a} - \frac{\dot{b}}{2b} + 3\dot{\psi} \right) - \frac{b}{a^2} \omega = 0 \quad (33)$$

and in the Lorentzian mode is

$$\ddot{\omega} + \dot{\omega} \left(\frac{\dot{a}}{a} - \frac{\dot{b}}{2b} + 3\dot{\psi} \right) + \frac{b}{a^2} \omega = 0 \quad (34)$$

Let we consider $\psi = 0$ case ⁶. It is easy to see that the first case can be deduced from the instanton condition

$$E_1^2 = H_1^2 \quad \text{or} \quad \frac{\omega}{a} = \pm \frac{\dot{\omega}}{\sqrt{b}} \quad (35)$$

The second equation (34) have not such reduction from the instanton condition (35) to this field equation. This is well known fact that the instanton can exist only in the Euclidean space. It allow us to say that the Euclidean equation (33) is simpler from the algorithmical point of view than the Lorentzian equation (34). In the first rough approximation we can suppose that the probability of the Euclidean mode is $p_{25}^+ = 1$ and consequently for the Lorentzian mode $p_{25}^- = 0$. Strictly speaking the exact definition for each p_{ab}^\pm probability should be [9]

$$p_{ab}^\pm = \frac{e^{-K_{ab}^\pm}}{e^{-K_{ab}^+} + e^{-K_{ab}^-}} \quad (36)$$

here K_{ab}^\pm is the AC for the $R_{ab}^\pm = 0$ equation. If $K_{25}^+ \ll K_{25}^-$ then we have $p_{25}^+ = 1$ and $p_{25}^- = 0$.

B. Fluctuation $R_{55}^+ \longleftrightarrow R_{55}^-$.

The R_{55} equation in the Euclidean mode is

$$\ddot{\psi} + \dot{\psi}^2 + \frac{\dot{a}}{a}\dot{\psi} + \frac{\dot{b}}{b}\dot{\psi} + \frac{r_0^2}{2}e^{2\psi} (H_1^2 + E_1^2) = 0, \quad (37)$$

and in the Lorentzian mode

$$\ddot{\psi} + \dot{\psi}^2 + \frac{\dot{a}}{a}\dot{\psi} + \frac{\dot{b}}{b}\dot{\psi} + \frac{r_0^2}{2}e^{2\psi} (-H_1^2 + E_1^2) = 0, \quad (38)$$

It is easy to see that the second equation (38) (Lorentzian mode) is much more simpler as it has the trivial solution

$$\psi = 0 \quad (39)$$

provided that we have the instanton condition $H_1^2 = E_1^2$. In the contrast with the previous subsection we see that in this case the Lorentzian mode is more preferable. As well in the contrast of the previous case we can suppose that in the first rough approximation the probability of the Euclidean mode of this equation is $p_{55}^+ = 0$ and consequently for the Lorentzian mode $p_{55}^- = 1$.

⁶below we will see that it will be in agreement with R_{55} equation

C. Remaining equations R_{11}^{\pm} and R_{22}^{\pm}

Let we write these equations in the following form

$$\frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{b}^2}{2b^2} - \sigma \frac{b}{a^2} + \frac{r_0^2}{b} \dot{\omega}^2 = 0, \quad (40)$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{2ab} + \sigma \left(-\frac{2}{a} + \frac{b}{a^2} \right) + \sigma \frac{r_0^2}{a^2} \omega^2 = 0, \quad (41)$$

In the Euclidean mode ($\sigma = +1$) can be $b = a$ (an isotropical Universe) and we have *only* one equation

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{2a^2} - \frac{1}{a} + \frac{r_0^2}{a^2} \omega^2 = 0. \quad (42)$$

In the Lorentzian mode ($\sigma = -1$) $b \neq a$ (this is a case of an nonisotropical Universe) and we have two equations

$$\frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{b}^2}{2b^2} + \frac{b}{a^2} + \frac{r_0^2}{b} \dot{\omega}^2 = 0, \quad (43)$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} - \left(-\frac{2}{a} + \frac{b}{a^2} \right) - \frac{r_0^2}{a^2} \omega^2 = 0, \quad (44)$$

We see that in the Lorentzian mode we have the nonisotropical Universe as the consequence of the presence of the magnetic field $H_1 = \omega/a$.

Certainly the one equation in the Euclidean mode is simpler than two equations in the Lorentzian mode. Our previous arguments according to the connection between the probability and the Kolmogorov's algorithmical complexity permit us to assume that in the first rough approximation the probability for (42) in the Euclidean mode is $p_{11}^+ = 1$ and consequently $p_{11}^- = 0$.

D. Mixed system of the equations

As the probability for each equations (32) are only $p = 0, 1$ we can write the *mixed* equation system for the Universe *fluctuated between Euclidean and Lorentzian modes*

$$\dot{\omega} = \pm \frac{\omega}{\sqrt{a}}, \quad (45)$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{2a^2} - \frac{1}{a} + \frac{r_0^2}{a^2} \omega^2 = 0. \quad (46)$$

here $b = a$ and $\psi = 0$. We are interested only for the case of the very Early Universe. For this conditions we have two regimes.

1. The mixed origin of the Universe

Hawking offers the following model of quantum birth of the Universe: at first appears a small piece of an Euclidean space (R^4 , S^4 or another smooth non-singular Euclidean space) then a Lorentzian Universe starts from a boundary of the initial Euclidean piece. In this scenario there is a hypersurface between Euclidean and Lorentzian spaces, see Fig.3.

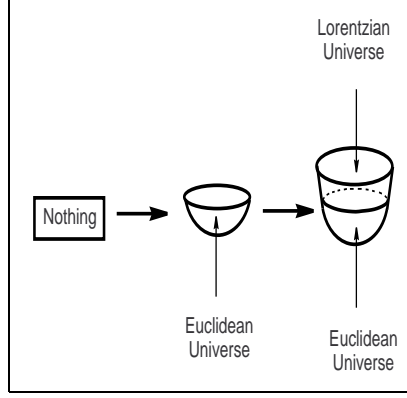


FIG. 3. An evolution of Lorentzian Universe from the “Nothing”.

In this section we assume that at first there is a quantum Universe fluctuated between Euclidean and Lorentzian modes and later happens a quantum transition to the Lorentzian mode.

We will search our metric in the following approximative form

$$a(t) = a_0 t^2 + a_2 t^4 + a_6 t^6 + a_8 t^8 + \dots, \quad (47)$$

$$\omega(t) = \omega_2 t^2 + \omega t^4 + \omega t^6 + \omega t^8 + \dots \quad (48)$$

After substitution to the initial mixed system of the equations (45)-(46) we have

$$a(t) \approx \frac{1}{4}t^2 - \frac{2}{9}r_0^2\omega_2^2t^4 - \frac{76}{405}r_0^4\omega_2^4t^6 - \frac{1856}{8505}r_0^6\omega_2^6t^8, \quad (49)$$

$$\omega(t) = \omega_2 t^2 + \frac{4}{9}r_0^2\omega_2^3t^4 + \frac{176}{405}r_0^4\omega_2^5t^6 + \frac{4673}{8505}r_0^6\omega_2^7t^8 \quad (50)$$

here ω_2 is the some constant. If we take the first term in (49) then the 4D metric is

$$ds^2 \approx dt^2 + t^2 d\Omega_3^2 \quad (51)$$

where $d\Omega_3^2 = \frac{1}{4}[(d\xi + \cos\theta d\varphi)^2 + (d\theta^2 + \sin^2\theta d\varphi^2)]$ is the metric of the 3D sphere S^3 . This metric is the flat R^4 Euclidean space.

If we take the first and second terms in (49) expression then we have

$$ds^2 \approx dt^2 + \left(\frac{1}{4}t^2 - \frac{2}{9}r_0^2\omega_2^2t^4\right) d\Omega_3^2 \quad (52)$$

and the Ricci scalar for this metric is

$$R \approx 32r_0^2\omega_2^2 + 96r_0^4\omega_2^4t^2 \quad (53)$$

Immediately we see that in the first approximation it is the 4D sphere S^4 .

The numerical calculations of the initial equations are presented on the Fig.4,5.

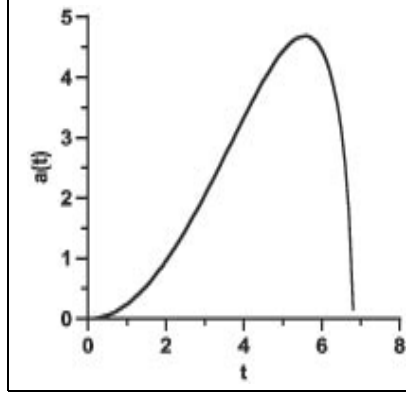


FIG. 4. The $a(t)$ function. $\omega_2 = 0.1$, $r_0 = 1.0$

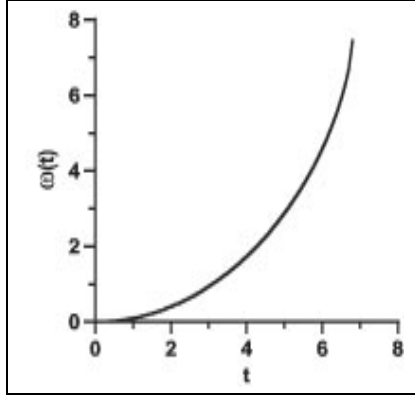


FIG. 5. The $\omega(t)$ function. $\omega_2 = 0.1$, $r_0 = 1.0$

For our model the ascending branch of the $a(t)$ function is important. We can assume that this piece of the fluctuating ⁷ non-singular Universe is born from "Nothing". Then in some moment a quantum transition to one mode (Lorentzian) takes place and simultaneously (or later) the G_{55} metric component become the non-dynamical variable. All this gives us the 4D Lorentzian Universe, see Fig.6.

⁷between Euclidean and Lorentzian modes

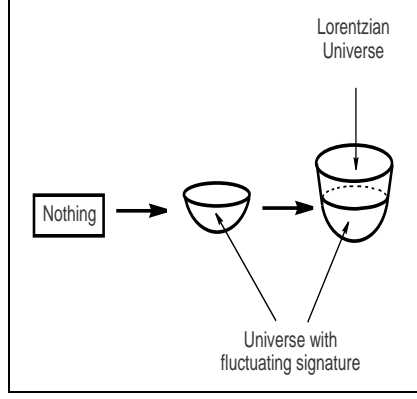


FIG. 6. An evolution of Lorentzian Universe through the non-singular mixed Universe with the fluctuating metric signature.

2. The bouncing Universe

The bouncing Universe means that at the origin ($t = 0$) the radius of the Universe has a minimum, i.e. $\dot{a}(0) = 0$ but $a(0) \neq 0$. The (45) equation show us that by this condition the Universe evolution can not be symmetrical relative to the origin ($t = 0$): if $a(t)$ is an even function then $\omega(t)$ and $\dot{\omega}(t)$ should be either even or odd functions simultaneously that can not be. Therefore the evolution of the Universe can be only nonsymmetrical relative to the point $t = 0$. At the origin ($t = 0$) the functions $\omega(t)$ and $a(t)$ can be expanded as follows

$$\omega(t) = \omega_0 + \omega_1 t + \dots, \quad (54)$$

$$a(t) = a_0 + a_2 \frac{t^2}{2} + \dots \quad (55)$$

The substitution in the (45)- (46) equations give us

$$\omega_1 = \pm \frac{\omega_0}{\sqrt{a_0}}, \quad a_2 = 1 - \frac{r_0^2}{a_0} \omega_0^2 \quad (56)$$

The numerical calculations of the (45)-(46) equations with (54), (55) initial conditions are presented on the Fig.7, 8.

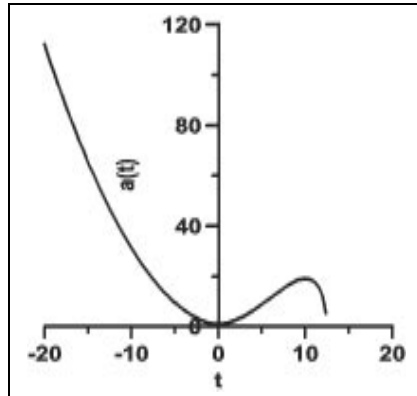


FIG. 7. The $a(t)$ function. $r_0 = 1.0, a_0 = 1.0, \omega_0 = 0.1$

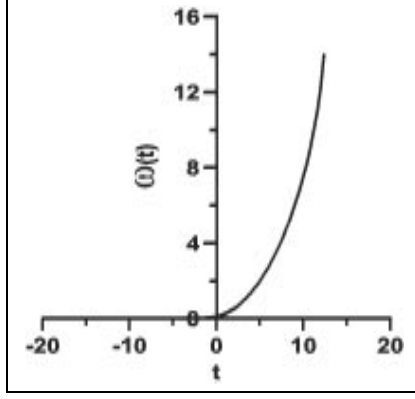


FIG. 8. The $\omega(t)$ function. $r_0 = 1.0, a_0 = 1.0, \omega_0 = 0.1$

The asymptotical behaviour for $t \rightarrow -\infty$ is

$$\omega(t) \approx \frac{\omega_\infty}{t^2}, \quad (57)$$

$$a(t) \approx \frac{t^2}{4} \quad (58)$$

This means that by $t \rightarrow -\infty$ we have a flat R^4 Euclidean space⁸ without the electromagnetic field and by $t > 0$ there is a Baby Universe with a singularity at some $t = t_\infty$. Certainly our approximation is correct only not far from origin $t = 0$ where $a \approx l_{Pl}^2$. We have to note that ω_0 can have an arbitrary value as the ω potential is dimensionless.

IV. PHYSICAL INTERPRETATION

For both cases in the (IIID 1) and (IIID 2) subsections the piece of the solution by $|t| \approx t_{Pl}$ can be interpreted as the quantum birth of the Universe with the fluctuating signature of metric. Roughly speaking, the 4D Universe is in the Euclidean mode but the 5th extra dimension is in the Lorentzian mode. As well we have the next interesting result: *the fluctuation of the metric signature leads to freezing the part of MD metric connected with the 5th dimension*. It is necessary to note that the R_{00} equation is not satisfy for this solution as it is a mixture of the Euclidean and Lorentzian modes. This is not so bad as in the Einstein equations one equation is the consequence of the others in view of the Bianchi identity

$$\left(R_B^A - \frac{1}{2} \delta_B^A R \right)_{;A} = 0 \quad (59)$$

i.e. all dynamic of the gravitational field is contained in the (27)-(30) equations and one remaining equation is an information about the initial conditions (in our case $R_{00} - \frac{1}{2} \eta_{00} R = 0$ equation have only the first time derivatives). That is we can say that the fluctuation of

⁸ R^4 is a section $\chi = const$ of the whole 5D spacetime

the metric signature leads to a quantum anomaly $R_{00} \neq 0$ or in the invariant form $R \neq 0$. Another words “quantum trembling” between Euclidean and Lorentzian metrics destroys the Bianchi identity.

V. DISCUSSION

In this paper we consider two examples of the nondifferentiable dynamic: the interchange of the sign between two components of the multidimensional metric and the quantum fluctuation between Euclidean and Lorentzian metrics. The existence of such kind of the nondifferentiable dynamic can lead to the interesting physical consequences: appearance of the spin-like structure on the throat of composite wormhole (this can be a possible connection with a superstring attached to two D-branes), freezing the physical degrees of freedom connected with the metric on the extra dimensions, quantum birth of the non-singular Universe with the quantum fluctuation of the metric signature between Euclidean and Lorentzian modes.

It is necessary to note that all these possibilities occur in a vacuum in the spirit of the Einstein idea that the right of the gravitational equations should be zero.

The Planck length paradigm says us that on this level the various unusual⁹ phenomena can occur. In the application to our case this permits us to say that the discrete (nondifferentiable) quantities can fluctuate on this level¹⁰. It is possible that the physical phenomena connected with the nondifferentiable dynamic can play very important role in the very Early Universe or on the level of the spacetime foam.

The basic idea presented by 't Hooft in Ref. [6] can be perceived so, that the fundamental states on the Planck level are the classical states and then the quantization should be stochastical: “... In our theory, quantum states are not the primary degrees of freedom. The primary degrees of freedom are deterministic states ...”. It is possible that the idea presented here have some connection with the 't Hooft stochastical quantization model. Actually, in the first case (section II) we have the quantum fluctuation between two classical states (between two metric signatures $\eta_{ab} = (\pm, -, -, -, \mp)$), in the second case we have the quantum fluctuating quantity σ in the classical Einstein equations that leads to the fluctuation between Euclidean and Lorentzian modes with the stochastical definition of probability according to the (36) equation.

VI. ACKNOWLEDGMENTS

This work is supported by a Georg Forster Research Fellowship from the Alexander von Humboldt Foundation. I would like to thank H.-J. Schmidt for the invitation to Potsdam Universität for research and D. Singleton for discussion.

⁹from the viewpoint of the non-Planckian physic

¹⁰we can name the dynamic of these quantities as the nondifferentiable dynamic.

-
- [1] M. Carfora and G. Ellis, Int. J. Mod. Phys. **D4**, 175 (1995).
 - [2] C. Hellaby, A. Sumeruk, and G. F. R. Ellis, Int. J. Mod. Phys. **D6**, 211 (1997).
 - [3] V. Dzhunushaliev, Gen. Relat. Grav. **30**, 583 (1998).
 - [4] V. Dzhunushaliev, Mod. Phys. Lett. A **13**, 2179 (1998).
 - [5] C. Misner and J. Wheeler, Ann. Phys. **2**, 525 (1957).
 - [6] G. 't Hooft, Class. Quant. Grav. **16**, 3263 (1999).
 - [7] A. Salam and J. Strathdee, Ann. Phys. **141**, 316 (1982).
 - [8] R. Percacci, J. Math. Phys. **24**, 807 (1983).
 - [9] V. Dzhunushaliev, Class. Quant. Grav. **15**, 603 (1998).
 - [10] A. Kolmogorov, *"Information Theory and Algorithm Theory: Logical Foundation of the Information Theory. Combinatorial Foundation of the Information Theory and the Probability Calculus"* (Nauka, Moscow, 1987).
 - [11] G. t'Hooft, Nucl. Phys. B **342**, 471 (1990).